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Development of optimal design formula for bi-tuned mass dampers using multi-objective optimization

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Abstract

In this paper the development of a closed-form, easy-to-use design formula for optimal control performance of bi-tuned mass dampers is described. First, optimal parameters of bi-tuned mass dampers are investigated by a multi-objective optimization technique for which two performance measures, termed as nominal and robust performance indices, are defined in terms of the maximum value of the frequency response function of the damped structure in its original design condition and that in perturbed conditions, respectively. Since the ratio of the mass of the bi-tuned mass dampers to that of the structure affects the control performance significantly, the multi-objective optimization process is repeated for various mass ratios. For each configuration, the plot of Pareto-optimal solutions in the objective function space exhibited a bifurcation point which could be used to improve the nominal or robust performances. Since the robust performance can be greatly improved up to the bifurcation point without a significant loss of the nominal performance, these bifurcation points are selected for the development of an optimal design formula. Simple closed-form expressions for such optimal tuning frequencies and damping ratios of bi-tuned mass dampers are then derived using a nonlinear curve-fitting technique. To verify the performance of the bi-tuned mass dampers system obtained by the proposed optimal design formula, illustrative examples are presented for a single- and bi-tuned mass dampers systems using the full-order model of the building structure. The nominal and robust performances of the optimal designs are examined through a parametric study on characteristics of ground motions. The results of a stochastic dynamic analysis demonstrate that the proposed design formula guarantees both nominal and robust performances of bi-tuned mass dampers systems in controlling the responses of the building structures under seismic excitations.

1. Introduction

It is well known that a tuned mass damper consisting of a mass, a spring and a damper is quite effective for vibration control of civil structures in many practical situations due to its simplicity and high level of reliability [1,2]. Remarkable advances have been made in optimal design formulas for single-tuned mass damper systems under various types of excitations [2–8]. For example, Den Hartog [2] demonstrated that the frequency

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response function of a harmonically excited structure with a single-tuned mass damper always passes two fixed points regardless of the damping ratio of the single-tuned mass damper and derived simple expressions for optimal parameters of the single-tuned mass damper in an analytical manner. However, the single-tuned mass damper system has a main drawback—its performance may worsen due to mis-tuned frequency or off-optimal damping [9]. A possible remedy to this is using more than one tuned mass damper with different dynamic characteristics. Accordingly, many researchers have studied how to determine the distribution of frequencies and damping ratios of the multiple-tuned mass dampers under various loading conditions, and suggested the corresponding optimal design formulas [9–15]. Among various types of multiple mass dampers, this paper focuses on two small tuned mass dampers, termed as bi-tuned mass dampers. This bi-tuned mass dampers system does not guarantee the existence of the fixed points that all the frequency response functions pass regardless of damping ratios of tuned mass dampers. Hence, a closed-form expression for optimal parameters of bi-tuned mass dampers cannot be obtained through an analytical approach such as Den Hartog's. As an attempt to help designers obtain optimal control performance in both original and off-tuning conditions using bi-tuned mass dampers systems, this paper derives a closed-form, easy-to-use design formula of optimal frequencies and damping ratios.

In order to achieve satisfactory performance of a bi-tuned mass dampers system, we first introduce two performance measures to quantify the effectiveness and robustness of the bi-tuned mass dampers in the original and off-tuning conditions, respectively. First, the "nominal" performance index is defined in terms of the peak response of the original structure subject to harmonic excitations. Second, the "robust" performance index is defined in terms of the peak response when the structure experiences perturbation in its dynamic properties due to normal wear or damage, or incomplete description of a real structure by a numerical model. By describing the performances as two different objective functions during the optimization process, the structure with the optimally designed bi-tuned mass dampers is expected to maintain excellent performance in its original condition with robustness against off-tuning events.

Since these two performances are both important and a change in the design does not necessarily improve or worsen both performances simultaneously, it is necessary to use an efficient optimization method to find parameters that reduce the peak responses both in original and perturbed conditions. A conventional approach is to use weighting factors to combine the multiple objectives into a single objective function. In the absence of further information or any preference on the multiple objectives, however, the single-objective optimization approach based on arbitrary weights does not guarantee a compromising solution for these multiple objectives. Therefore, we adopt a multi-objective optimization approach [16,17] to maximize the nominal and robust performances simultaneously without prescribing the weighting factors arbitrarily.

Since improvement in one objective leads to degradation in at least one of the remaining objective functions, the presence of multiple objectives in a problem, in principle, produces a set of optimal solutions, often called Pareto-optimal solutions. Accordingly, the multi-objective optimization approach guides the multi-point searching process toward a uniformly spread-out Pareto-optimal front in multi-dimensional objective space. Thus, the distribution of the Pareto-optimal solutions well describes the relative importance between the multiple objectives, or how the multiple objectives are competent with each other in the objective space. This descriptive information on the multiple objectives enables us to obtain a reasonable solution that achieves at least similar level of nominal performance to single-tuned mass damper and as much robust performance as possible. The ratio of the total mass of the bi-tuned mass dampers to that of the structure affects the control performance of the damped structural system. Thus, we repeat the multi-objective optimization process as the mass ratio of the bi-tuned mass dampers is varied, and a set of reasonable optimal solutions are selected for the given range of mass ratios. Finally, a nonlinear curve fitting scheme is utilized to derive a closed-form optimal design formula of bi-tuned mass dampers as a function of the mass ratio.

In order to demonstrate the performance of the proposed design formula of the bi-tuned mass dampers, we apply the optimally designed bi-tuned mass dampers system to a full-order model of the structural system, and its nominal and robust performances are investigated for a wide range of parameters representing the intensity, dominant frequency and bandwidth of ground motion excitations. It is also compared to the performances of an single-tuned mass damper designed by Den Hartog design formula which is the most commonly used design formula of the single-tuned mass damper in its practical applications.

2. Nominal and robust performance measures of bi-tuned mass dampers system

2.1. Structural model of bi-tuned mass dampers system

Consider a primary structure with a bi-tuned mass damper system installed. The primary structure is modeled as a single-degree-of-freedom oscillator that represents the structural mode to be controlled. In general, the first modal frequency is selected as the target frequency to be controlled since the contribution of the first modal behavior to the global response is dominant in most civil structures. Hence, the mass (m_s), damping coefficient (c_s) and stiffness (k_s) of the single-degree-of-freedom model of the primary structure correspond to those of the first mode of the structure. Each of the two tuned mass dampers is also modeled as an single-degree-of-freedom system, and their dynamic properties are characterized by mass m_j, damping coefficient c_j and stiffness k_j ($j = 1,2$). In this paper, we assume that the masses of the two tuned mass dampers are the same, i.e., $m_1 = m_2$. When the two tuned mass dampers are attached to the primary structure as shown in Fig. 1, the equation of motion is

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t), \tag{1}$$

where $\ddot{\mathbf{x}}(t)$, $\dot{\mathbf{x}}(t)$ and $\mathbf{x}(t)$, respectively, denote (3×1) dimensional vectors of accelerations, velocities and displacements of the structural system; the vector $\mathbf{x}(t)$ consists of the displacements of the structure and the two tuned mass dampers relative to the base, i.e., $\mathbf{x}(t) = [x_s(t), x_1(t), x_2(t)]^{\mathrm{T}}$; $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{f}(t)$, respectively, denote the mass, damping and stiffness matrices and the force vector. More specifically,

$$\mathbf{M} = \begin{bmatrix} m_s & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_2 \end{bmatrix}, \tag{2a}$$

$$\mathbf{C} = \begin{bmatrix} c_s + c_1 + c_2 & -c_1 & -c_2 \\ -c_1 & c_1 & 0 \\ -c_2 & 0 & c_2 \end{bmatrix}, \tag{2b}$$

$$\mathbf{K} = \begin{bmatrix} k_s + k_1 + k_2 & -k_1 & -k_2 \\ -k_1 & k_1 & 0 \\ -k_2 & 0 & k_2 \end{bmatrix}, \tag{2c}$$

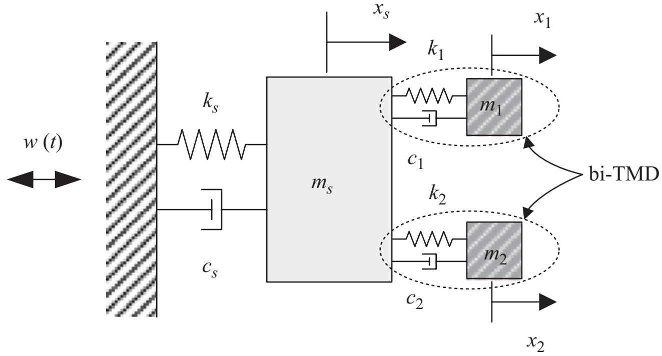

Fig. 1. Structural model of a structure with bi-tuned mass dampers (bi-TMD).

$$\mathbf{f}(t) = -\mathbf{M}\underset{\sim}{\mathbf{1}}\,w(t) = \begin{bmatrix} -m_s \\ -m_1 \\ -m_2 \end{bmatrix} w(t), \tag{2d}$$

in which $w(t)$ is the acceleration of the base-excitation.

In order to obtain the frequency response function of the structural response with respect to the excitation $w(t)$, we describe the equation of motion in Eq. (1) by a state-space representation [18]

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}w(t), \tag{3}$$

where $\mathbf{z}(t)$ is the state vector; $\mathbf{A}$ is the system matrix; $\mathbf{B}$ is the input matrix; and more specifically,

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix}, \tag{4a}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \tag{4b}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ -\underset{\sim}{\mathbf{1}} \end{bmatrix}, \tag{4c}$$

where $\mathbf{I}$ is the identity matrix and $\underset{\sim}{\mathbf{1}}$ is a vector of ones. A generic output vector $\mathbf{y}(t)$ is often described in terms of $\mathbf{z}(t)$ and $w(t)$, i.e.,

$$\mathbf{y}(t) = \mathbf{C_y}\mathbf{z}(t) + \mathbf{D_y}w(t), \tag{5}$$

in which $\mathbf{C_y}$ and $\mathbf{D_y}$ are the output matrices. In the case where the displacement of the primary structure, $x_s(t)$ is of interest, for example, $\mathbf{C_y} = [1\ \ 0\ \ \ldots\ \ 0]$ and $\mathbf{D_y} = [0]$.

According to the theory of linear system [18], the frequency response function is given by

$$h_s(\omega) = \mathbf{C_y}(\mathrm{i}\omega \cdot \mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D_y}, \tag{6}$$

where ω is the circular frequency argument.

2.2. Nominal and robust performance indices

This study aims to develop a closed-form optimal design formula for bi-tuned mass dampers system that guarantees not only effective performance in original (or exact tuning) condition but also robust performance against a certain range of possible errors or mis-tuning in structural parameters used for optimization. For this purpose, we first introduce two performance measures for a bi-tuned mass dampers system, i.e., ''nominal'' and ''robust'' performance indices. The nominal performance index of a bi-tuned mass dampers, J_N quantifies the effectiveness of the bi-tuned mass dampers performance when it is precisely tuned to the primary structure. Thus, J_N is defined as the peak value of the frequency response function of the original structure with a bi-tuned mass dampers system (i.e., when its actual frequency is exactly the same as the tuning frequency of the bi-tuned mass dampers) over a range of exciting frequency, i.e.,

$$J_N = \max_{\omega \in \Omega} |h_s(\omega)|, \tag{7}$$

where Ω denotes a range of possible exciting frequency ω.

Along with the definition of the nominal performance index, the robust performance index, J_R is defined by the maximum value of the frequency response function of the perturbed structure with a bi-tuned mass dampers system. The perturbed condition of the structure may arise from changes in dynamic properties of the main structure due to normal wear or damage, or incomplete description of a real structure by a numerical model. The robust performance index thus accounts for performance degradation of the bi-tuned mass dampers under the presence of uncertain structural properties and can be expressed as

$$J_R = \max_{\delta k_s \in \Psi} [\max_{\omega \in \Omega} |h_\Delta(\omega, \delta k_s)|], \tag{8}$$

where the frequency perturbation of the structure is modeled by stiffness variation of the primary structure, δk_s so that the influence of possible variations of other properties such as structural mass or damping on the frequency perturbation are reflected indirectly; $h_\Delta(\omega, \delta k_s)$ is the perturbed frequency response function due to the stiffness variation δk_s; and Ψ is a range of possible stiffness variations. For the given stiffness variation, the peak value of the perturbed frequency response function is at first calculated over a range of exciting frequency ω, and the robust performance index is calculated as the maximum value of the peak frequency response function over the range of stiffness variations δk_s. Therefore, this robust performance index always corresponds to the maximum frequency response of the primary structure even if the maximum frequency response does not occur at the maximum perturbation.

The smaller value of performance indices indicates better performance of the bi-tuned mass dampers. Hence, the bi-tuned mass dampers system with the smallest value of J_N is most effective in reducing the maximum displacement of the original structure, whereas the bi-tuned mass dampers system with the smallest value of J_R is most effective in reducing the possible maximum displacement of the perturbed structure within a certain range of frequency perturbations.

3. Multi-objective optimization approach

The main goal of the proposed optimization approach is to find a bi-tuned mass dampers system that guarantees both effective and robust performances by optimization. Conventional optimization approaches often combine such multiple objective functions into a single-objective function by prescribing arbitrary importance weights. However, these two objectives do not necessarily diminish at the same time and the relative importance of different objectives is not known a priori in most cases. Therefore, we hereby adopt a weighting-free optimization approach for simultaneous minimization of the multiple objectives. Thus, the optimization problem is formulated using a vector form of objective function such that

$$\begin{aligned} &\underset{\mathbf{x}=\{\omega_1,\zeta_1,\omega_2,\zeta_2\}}{\text{minimize}} \quad \begin{Bmatrix} J_N \\ J_R \end{Bmatrix} \\ &\text{subject to} \quad \delta k_s^- \leqslant \delta k_s \leqslant \delta k_s^+, \quad \omega \in \Omega, \end{aligned} \tag{9}$$

where ω_j and ζ_j ($j = 1,2$) correspond to the natural frequencies and damping ratios of the tuned mass dampers to be determined; and δk_s^- and δk_s^+ are the expected bounds on the perturbed frequency of the structure.

The presence of multiple objectives in a problem, in principle, produces a set of optimal solutions, often called as Pareto-optimal solutions. This is due to the fact that each point in the Pareto-optimal solution surface is optimal in the sense that improvement in one objective function leads to degradation in at least one of the remaining objective functions. Since none of the solutions in the Pareto-optimal set is absolutely better than any other, any one of them is an acceptable solution, which leads to a set of non-dominated optimal solutions. Since classical optimization methods work with a single solution in each iteration, they are required to be applied repeatedly, hopefully finding one distinct Pareto-optimal solution each time. On the other hand, genetic algorithm [19] can work with a population of multiple points and can capture a set of Pareto-optimal solutions through a single run of genetic algorithm. This aspect makes genetic algorithm well-suited to solve multi-objective optimization problem for finding multiple Pareto-optimal solutions while saving significant computational time. Accordingly, a number of different multi-objective genetic algorithm implementations [16,17,20–28] have been presented so far. Of particular interest among these is a fast elitist non-dominated sorting genetic algorithm [22] in this study, which is an improved version of non-dominated sorting genetic algorithm [21] and often called non-dominated sorting genetic algorithm-II.

Fig. 2 illustrates the procedures of the non-dominated sorting genetic algorithm-II technique (for more details of the algorithm, see [22,29]). The non-dominated sorting genetic algorithm-II first generates a population consisting of N "individuals" which are randomly distributed in a solution space as shown in

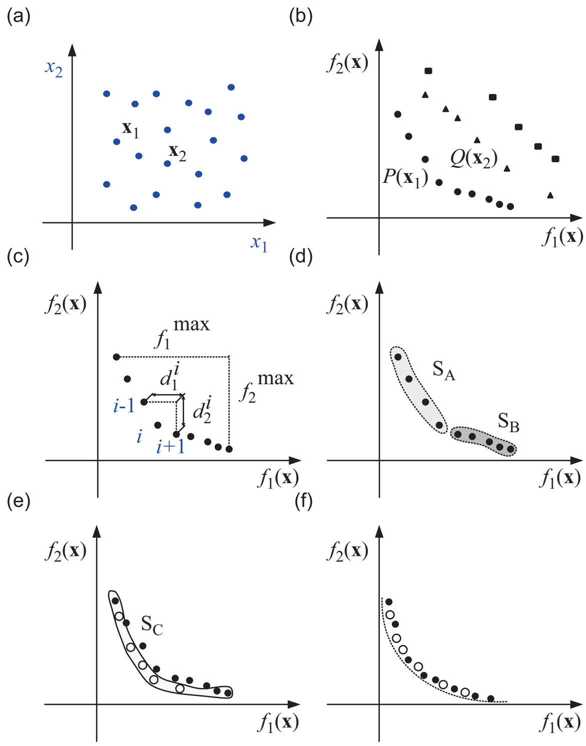

Fig. 2. Schematic representation of non-dominated sorting genetic algorithm-II (NSGA-II): (a) random generation of N chromosomes in solution space; (b) rank sorting of N individuals in objective space; (c) evaluation of the crowding distance; (d) fitness assignment based on the rank and the crowding distance; (e) elitism; and (f) evolution toward global Pareto front. ●: Solutions with rank 1; ▲: solutions with rank 2; and ■: solutions with rank 3. f_j^{max}: maximum distance along with j-th objective function; d_k^i: distance between two solutions adjacent to i-th individual along with the k-th objective function; S_A: a set of solutions with the same rank and larger crowding distance; S_B: a set of solutions with the same rank and smaller crowding distance; ○: newly generated solutions; and S_C: a set of new solutions with rank 1 after comparing newly generated solutions with previous rank 1 solutions.

Fig. 2(a). Each individual corresponds to a set of the design variables and is expressed in a form of "chromosome". In this study, the dynamic properties of bi-tuned mass dampers such as natural frequencies ω_j and damping ratios ζ_j ($j = 1,2$) are encoded in sequence into one chromosome. The generated population of size N is used to evaluate the objective functions in Eq. (9) using Eqs. (7) and (8).

Based on the objective function values, non-dominated sorting genetic algorithm-II ranks the N individuals using the "non-domination" concept. The non-domination concept is as follows; an individual $\mathbf{x}_1$ is said to dominate $\mathbf{x}_2$ if both of the following conditions are satisfied: (1) $\mathbf{x}_1$ is no worse than $\mathbf{x}_2$ in all objectives and (2) $\mathbf{x}_1$ is strictly better than $\mathbf{x}_2$ in at least one objective. For example, in Fig. 2(b), an individual $\mathbf{x}_1$ is said to dominate $\mathbf{x}_2$ since $f_1(\mathbf{x}_1) < f_1(\mathbf{x}_2)$ and $f_2(\mathbf{x}_1) < f_2(\mathbf{x}_2)$. Using the above concept, individuals in the population are classified into groups of different non-domination levels. All the individuals which are not dominated by any others in the population are assigned to "1" or the first rank, and constitute the first front in the objective-function space. For the remaining individuals, this procedure is applied again to find the individuals of the second rank. Therefore, the individuals in the second front are dominated by the individuals in the first front only. This sorting process continues to assign the consecutive ranks to the remaining individuals till all the population members are classified into their corresponding non-domination levels. Fig. 2(b) illustrates the ranked individuals in the objective function space. All the individuals in each front share the same fitness value which is equal to the non-domination level, or the front they belong to. Thus, the individuals in the first front turn out to be Pareto-optimal solutions.

In addition to the fitness assignment, it is also important to maintain population diversity in the current non-dominated front. For that purpose, a density-estimation metric, so-called crowding distance, is defined by

$$\text{crowding distance} = \sum_{k=1}^{m} \frac{d_k^i}{f_k^{\max}}, \tag{10}$$

where as shown in Fig. 2(c), $f_k^{\max}$ is the maximum difference of the k-th objective function between two outmost individuals in the Pareto front; $d_k^{\,i}$ is the distance between two individuals adjacent to i-th individual along with k-th objective; m is the number of the objective functions of interest. Thus, the crowding distance is calculated for each individual through the summation of the multi-dimensional distances between two adjacent individuals along the multi-objective space. Each of the multi-dimensional distances is normalized by the maximum distance in each objective. This metric serves as an estimate of the density of solutions surrounding a particular solution in the front.

Once the individuals are ranked and their crowding distances are assigned, the binary tournament selection is operated based on the fitness value and crowding distance. non-dominated sorting genetic algorithm-II at first selects parents with the lowest rank from the current-generation population. If the two individuals belong to the same front, their crowding distances are compared and the individual with larger crowding distance is selected as a parent. As shown in Fig. 2(d), the solution set S_A is selected as a parent with higher possibility than the solution set S_B because both solution sets have the same rank but S_A has larger crowding distance than S_B. Due to this selection process, the optimal solutions obtained by non-dominated sorting genetic algorithm-II are uniformly distributed along with the current non-dominated front. The selected parents generate the corresponding offsprings through the crossover and mutation operators [19]. As shown in Fig. 2(e), the newly generated offspring population is combined with the current generation population, and an elitism process is applied to the combined population in order to preserve the best solutions, which is operated by two criteria, i.e., the ranks and crowding distances of the individuals. The best solutions of the current generation constitute the individuals of the next generation. Then, all the above processes such as the binary tournament selection, generation of offspring, crossover and mutation operators are iterated up to the specified maximum number of generation. Finally, as shown in Fig. 2(f), the non-dominated sorting genetic algorithm-II guides multiple individuals towards a global Pareto front, while maintaining the diversity of the solutions along the Pareto front in the multi-objective space, which herein correspond to a set of solutions with smaller values in both sides of nominal and robust performances.

4. Development of optimal design formula for bi-tuned mass dampers

As already mentioned in Introduction, the mass ratio of the bi-tuned mass dampers affects the control performance. Therefore, the proposed optimization approach is repeated while varying the mass ratio of the bi-tuned mass dampers. Accordingly, a total of seven different mass ratios of the bi-tuned mass dampers, {0.5, 1.0, 2.0, 3.0, 4.0, 5.0, 10.0 percent} are considered to cover the entire range of practical applications, 0.5–10 percent [11]. When evaluating the robust performance measure, the stiffness of the primary structure is varied from 85 to 115 percent of the nominal value. During optimization by non-dominated sorting genetic algorithm-II, a total of 100 chromosomes are generated each time and the generations are repeated 5000 times. Then, the optimal frequencies and damping ratios of bi-tuned mass dampers, ω_j and ζ_j ($j = 1,2$) are explored in a range of 60–110 percent of the first modal frequency and 2–20 percent of critical damping, respectively.

Fig. 3 illustrates the optimization results of a bi-tuned mass dampers in the space of the two objective functions for the seven mass ratios considered. It displays seven different Pareto-optimal fronts, each of which consists of 100 Pareto-optimal solutions for a given mass ratio. In each Pareto front, the obtained Pareto-optimal solutions are well distributed, and they indicate an inversely proportional relationship between the nominal and robust performance indices. Note that smaller values of the two indices indicate the greater reduction on the structural responses under the original and perturbed conditions, respectively. Thus, all the obtained Pareto-optimal solutions are optimal in the sense that an increase in the nominal performance measure leads to a decrease in the robust performance measure. It is noteworthy that each Pareto front has a bifurcation point, as indicated by several marker points in Fig. 3. Since the robust performance can be greatly improved up to the bifurcation point without a significant loss of the nominal performance, we can choose this

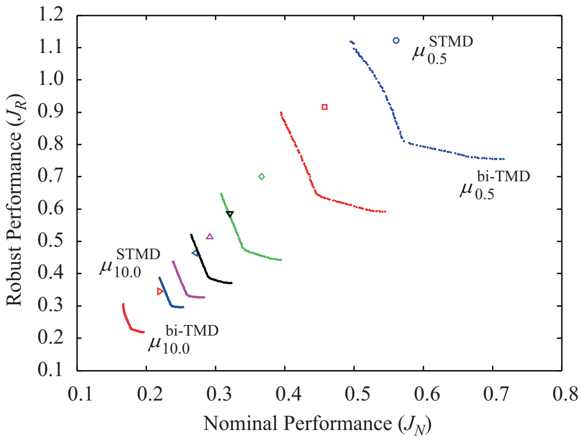

Fig. 3. Pareto optimal solutions (bi-tuned mass dampers—bi-TMD) and Den Hartog optimal solutions (single-tuned mass damper—STMD) for selected mass ratios. Each marker denotes the optimal solution of single-tuned mass damper by Den Hartog formula: ○: $\mu_{0.5}^{\text{STMD}}$, i.e., $\mu = 0.5$ percent (single-tuned mass damper); □: $\mu = 1.0$ percent (single-tuned mass damper); ◇: $\mu = 2.0$ percent (single-tuned mass damper); ∇: $\mu = 3.0$ percent (single-tuned mass damper); △: $\mu = 4.0$ percent (single-tuned mass damper); ◁: $\mu = 5.0$ percent (single-tuned mass damper); and ▷: $\mu_{10.0}^{\text{STMD}}$, i.e., $\mu = 10.0$ percent (single-tuned mass damper). Dotted lines represent a set of Pareto optimal solutions of bi-tuned mass dampers; $\mu_{0.5}^{\text{bi-TMD}}$ $\mu = 0.5$ percent (bi-tuned mass dampers) and $\mu_{10.0}^{\text{bi-TMD}}$ $\mu = 10.0$ percent (bi-tuned mass dampers).

point among the Pareto solution set as a reasonable optimal design solution that achieves both nominal and robust performances. The proposed multi-objective optimization approach enabled us to identify this optimal design point in an efficient manner. For comparison of optimal performances, the single-tuned mass damper systems with the same mass ratios designed by Den Hartog formula are also presented by seven different marker points. Here, the same mass ratio means that the mass ratio of single-tuned mass damper is equal to the sum of the two mass ratios of the bi-tuned mass dampers. All the bifurcation points show a great enhancement in robust performance, yet their nominal performance indices remain still smaller than those of the single-tuned mass damper by Den Hartog formula except for a mass ratio of 0.5 percent where the nominal performance index of bi-tuned mass dampers is slightly larger than that of single-tuned mass damper. Even if the bifurcation point for the case of 0.5 percent mass ratio shows slightly poorer nominal performance, it does not necessarily mean that the bi-tuned mass dampers system with the small mass ratio is always poorer than the single-tuned mass damper in the nominal performance. As seen in Fig. 3, there still exist many Pareto solutions which are better than single-tuned mass damper in terms of both nominal and robust performances even when the mass ratio is very small. It is also observed that the difference between the nominal performances of the single-tuned mass damper and bi-tuned mass dampers systems is insignificant when the mass ratio is very small. With the minor sacrifice of the nominal performance, the robust performance of the bi-tuned mass dampers can be greatly improved. Therefore, we decided to use the bifurcation points for developing optimal design formulas of the bi-tuned mass dampers. Therefore, the optimal solutions corresponding to the bifurcation points can guarantee improved robust performance while maintaining the nominal performance similar to or better than that of the single-tuned mass damper designed by Den Hartog formula.

Due to the randomness in generating searching points in genetic algorithm, there exist some variations in the obtained optimal solutions in general even for the same parameters given. Therefore, we select five Pareto-optimal solutions around each of the seven bifurcation points and use them to derive a design formula. Fig. 4 illustrates the optimal parameters for a total of $5 \times 7 = 35$ bi-tuned mass dampers systems, which are denoted by markers '□' and '◇'. Note that dotted lines with a marker '○' are the optimal parameters of single-tuned mass damper by Den Hartog design formula. It is observed in Fig. 4 that one of the optimal frequency ratios of the bi-tuned mass dampers is distributed close to 1.0, which means its frequency is tuned near to the first modal frequency of the primary structure. Also note that the optimal frequency ratios decrease and the corresponding optimal damping ratios increase as the mass ratio increases. These parameters are used in a

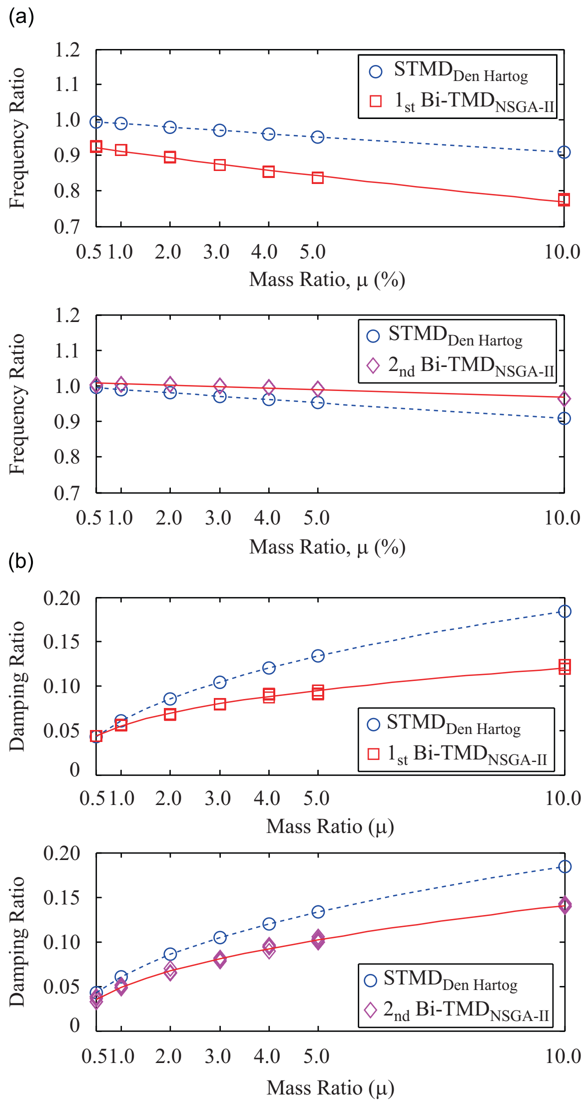

Fig. 4. Optimal parameters of STMD (single-tuned mass damper) by Den Hartog formula (○) and of bi-TMD (bi-tuned mass dampers) by NSGA-II (non-dominated sorting genetic algorithm-II) (□): (a) optimal frequency ratio and (b) optimal damping ratio.

nonlinear curve-fitting scheme [30] to derive closed-form expressions of the optimal frequencies and damping ratios of a bi-tuned mass dampers for given mass ratio μ.

In order to make simple and easy-to-use design formula for a bi-tuned mass dampers, we adopt the following form similar to Den Hartog design formula [2] for frequency ratio and damping ratio:

$$f_{\mathrm{opt}} = \frac{1}{\alpha + \beta \times \mu}, \tag{11a}$$

$$\zeta_{\mathrm{opt}} = \left(\frac{\varepsilon + \phi \times \mu}{\chi + \delta \times \mu}\right)^{\gamma}. \tag{11b}$$

As for the frequency ratio f_{opt}, Eq. (11a) gives a close approximation to the optimal frequency values given in Fig. 4(a). On the other hand, the coefficient δ in Eq. (11b) approaches to zero when Eq. (11b) is fitted to the

Table 1
Design formulas of single- and bi-tuned mass dampers.

Design formulas	Frequency ratio	Damping ratio
Den Hartog	$\dfrac{1}{1+\mu}$	$\sqrt{\dfrac{3\mu}{8(1+\mu)}}$
This study		
First bi-tuned mass dampers	$\dfrac{1}{1.0737+2.2593\cdot\mu}$	$0.2623\times\mu^{0.3386}$
Second bi-tuned mass dampers	$\dfrac{1}{0.9895+0.4418\cdot\mu}$	$0.4054\times\mu^{0.4600}$

optimal damping ratios in Fig. 4(b). Thus, the following simpler form is used to fit the curve in Fig. 4(b):

$$\zeta_{\text{opt}} = (\varepsilon' + \phi' \times \mu)^{\gamma'}. \tag{12}$$

Again, the coefficient ε' turns out to be close to zero, so it is dropped from Eq. (12). Finally, the closed-form expressions for the optimal frequency ratio and damping ratio of bi-tuned mass dampers are given in Table 1. The optimal parameters for the derived formula are depicted as solid lines in Fig. 4, which has a maximum relative error of 0.43 percent in frequency ratio and 2.78 percent in damping ratio. The magnitude of error for optimal damping ratio is relatively larger than that for optimal frequency ratio, which is caused by using a simple form of design formula and neglecting the corresponding insignificant error.

In order to verify the accuracy and effectiveness of the proposed design formula, the nominal and robust performances of the single-degree-of-freedom primary structure with single-tuned mass damper or bi-tuned mass dampers subject to harmonic excitations are evaluated using the optimal design formulas, i.e., Den Hartog design formula and the ones proposed in this study. Fig. 5 depicts the simulated results of the two damper systems. The horizontal axis corresponds to the mass ratio of single-/bi-tuned mass dampers. For bi-tuned mass dampers, the mass ratio is given in terms of the total mass of the two dampers. The vertical axis presents the nominal and robust performance indices (J_N, J_R) of the damped systems divided by those of the undamped system, i.e., the original or perturbed structure with no damper which are denoted by ${J_N}^0$ and ${J_R}^0$. On the whole, the performances of both single- and bi-tuned mass dampers are enhanced as the mass ratios of the dampers are increased, as observed also in Fig. 3. When the mass ratio is smaller than 1 percent, the nominal performance of bi-tuned mass dampers is slightly poorer than that of single-tuned mass damper whereas the robust performance of bi-tuned mass dampers remains better than single-tuned mass damper. As already mentioned before, this is due to the fact that the bifurcation points in Fig. 3 are selected for the derivation of the optimal design formula. One can further improve the design formula for bi-tuned mass dampers system such that it guarantees improvement over single-tuned mass damper system in terms of both nominal and robust performances by choosing Pareto solutions other than bifurcation points. However, this improvement is minimal and limited to the cases of small mass ratios. For a range of more than 1 percent of mass ratio, the bi-tuned mass dampers further improve the nominal performance over single-tuned mass damper while maintaining absolutely better robustness than single-tuned mass damper. Thus, these comparative results demonstrate that the proposed design formula can further improve the robust performance of the damped structural system while maintaining at least similar level of nominal performance to the single-tuned mass damper system. Moreover, the Pareto-optimal solutions selected as reasonable solutions in Fig. 3 are also presented for the purpose of verifying the accuracy of the proposed design formula. The difference between the performance indices of the optimal bi-tuned mass dampers systems obtained by non-dominated sorting genetic algorithm-II and those by the simple optimal design formula in Table 1 are insignificant. This confirms that the proposed simple design formula guarantees the optimal performance of bi-tuned mass dampers identified by the proposed multi-objective optimization approach accounting for both original and perturbed conditions.

The perturbation of frequency and damping ratio of the primary structure influences the optimal parameters of the bi-tuned mass dampers. In the proposed design approach, the effect of the frequency perturbation is incorporated into the robust performance index. However, the variation of the damping ratio

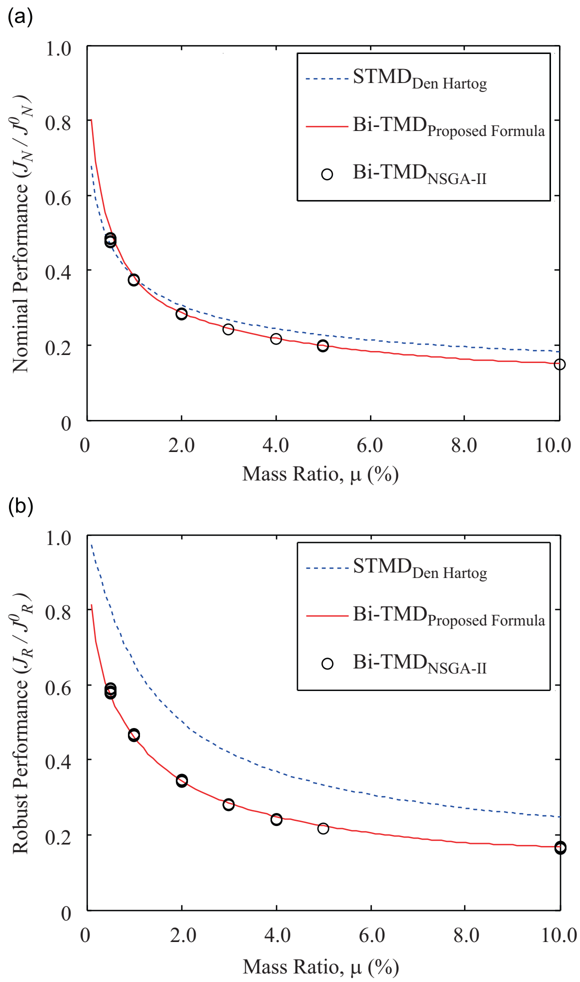

Fig. 5. Performance evaluation of STMD (single-tuned mass damper) and bi-TMD (bi-tuned mass dampers) with respect to mass ratio: (a) nominal performance and (b) robust performance. ⋯: Single-tuned mass damper by Den Hartog formula; —: bi-tuned mass dampers by proposed formula; and ○: bi-tuned mass dampers by NSGA-II (non-dominated sorting genetic algorithm-II).

of the primary structure is not considered. Since the damping ratio of the primary structure is also uncertain, its effect on the nominal and robust performances of the bi-tuned mass dampers needs to be examined in order to verify the applicability of the proposed design formula. Thus, the nominal and robust performances of the damper-controlled primary structure with different damping ratios are evaluated where the single-tuned mass damper is designed by Den Hartog design formula and the bi-tuned mass dampers are designed by the formulas proposed in this study. Fig. 6 depicts the simulated results of the two damper systems when the damping ratios of the primary structure are varied from 0.5 to 6 percent. In order to demonstrate the suitability of the proposed design formula for illustration purpose, the ratios of the total mass of single- or bi-tuned mass dampers to that of the primary structure are chosen as 1 and 3 percent. As for the single- and bi-tuned mass dampers with 1 percent mass ratio, their nominal performances are almost similar to each other. In contrast, the robust performance of the bi-tuned mass dampers is significantly better than that of the single-tuned mass damper, especially when the damping ratio of the primary structure is low. Especially when the damping ratio of the primary structure is 0.5 percent, the ratio of the robust performance measure of the single-tuned mass damper system to the undamped one exceeds 1.0. This indicates that the response controlled by the single-tuned mass damper is larger than the uncontrolled response or the response of the building with no dampers. However, the ratio of the robust performance measure of the bi-tuned mass dampers remains smaller than 1.0 even for the damping ratio 0.5 percent. This result demonstrates that the performance of the

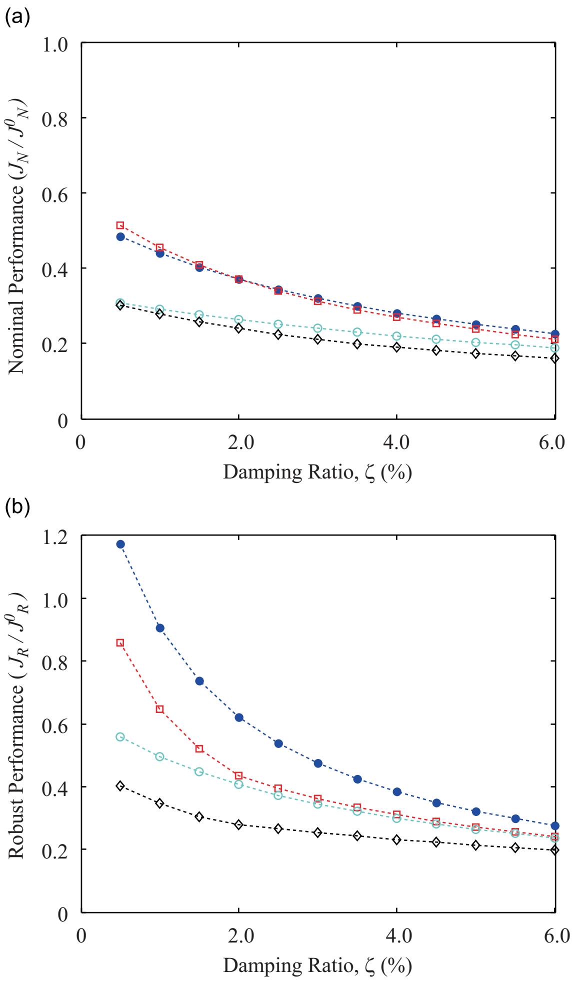

Fig. 6. Performance evaluation of single-tuned mass damper and bi-tuned mass dampers with respect to damping ratio of the primary structure: (a) nominal performance and (b) robust performance. ●: Single-tuned mass damper with 1 percent mass ratio by Den Hartog formula; □: bi-tuned mass dampers with 1 percent mass ratio by proposed formula; ○: single-tuned mass damper with 3 percent mass ratio by Den Hartog formula; ◇: bi-tuned mass dampers with 3 percent mass ratio by proposed formula.

single-tuned mass damper may be very sensitive to the variation of the damping ratio of the primary structure while the bi-tuned mass dampers shows more robustness against the off-tuning of the damping ratio. As the mass ratio of the single- or bi-tuned mass dampers is increased up to 3.0 percent, similar tendency is observed for both nominal and robust performances. The robust performance is greatly improved again by the bi-tuned mass dampers, while the nominal performances of the single- and bi-tuned mass dampers are almost similar to each other. It is noteworthy that the overall difference between the performance measures of the single- and bi-tuned mass dampers systems is decreased as the damping ratio of the primary structure is increased. This is because, as the damping ratio of the primary structure is increased, its role in controlling the structural responses becomes dominant and the damper system becomes less important. It is concluded that the proposed design formula can help achieve robust performance especially for the structures with small damping ratios.

5. Performance evaluation of optimal bi-tuned mass dampers for multi-degree-of-freedom structures

To confirm the applicability and the optimal performance of the proposed design approach in practical situations, the proposed design formula is applied to a primary structure with full-order models (i.e., not an

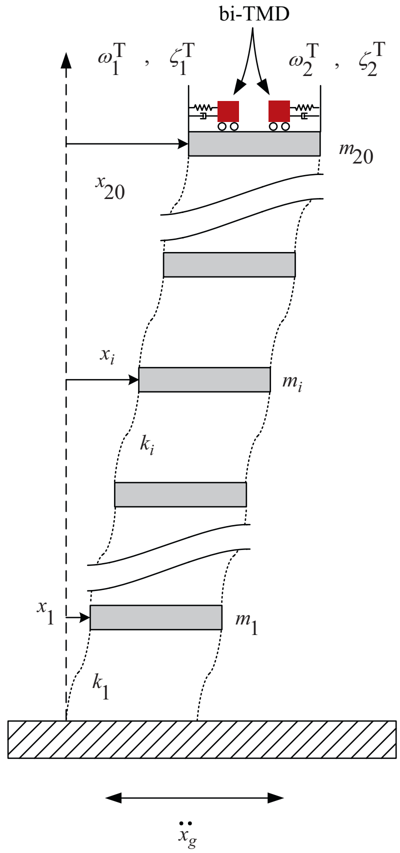

Fig. 7. Example 20-storey building with bi-TMD (bi-tuned mass dampers).

single-degree-of-freedom model representing the first mode only) of two 20-storey buildings that have different modal frequencies. In order to account for the stochastic nature of ground motions and its significant impact on the structural responses, we estimate the peak response of the damped structure by stochastic dynamic analysis. The two buildings have different inter-storey stiffness, 5600 and 22,400 MN/m, respectively. The mass of each floor is 800 ton for both buildings. A classical damping matrix with 2 percent of the critical damping for each mode is assumed for both buildings. Thus, the first modal frequencies of the two buildings turn out to be 0.51 Hz (building "A") and 1.02 Hz (building "B"), respectively. In order to demonstrate the good performance of the proposed design formula for a practical range of mass ratio, the total mass ratios of the bi-tuned mass dampers for the two buildings are chosen as 1 and 3 percent of the first modal mass of the corresponding structures. For the purpose of comparison, single-tuned mass damper designed by Den Hartog formula is also considered. All the damper systems are installed on the top floor of the buildings to suppress the seismic responses of the building, as shown in Fig. 7. The random seismic excitation is modeled as a zero-mean stationary, filtered white-noise process defined by the following Kanai–Tajimi power spectral density (PSD) function [31]

$$\Phi_{\ddot{x}_g\ddot{x}_g}(\omega) = \frac{\omega_g^4 + 4\zeta_g^2\omega_g^2\omega^2}{(\omega_g^2 - \omega^2)^2 + 4\zeta_g^2\omega_g^2\omega^2} \times \Phi_0 \tag{13}$$

where ω_g is the predominant frequency, ζ_g is the parameter controlling the bandwidth, and Φ_0 is the parameter for the seismic intensity, respectively.

Of our particular interest is the peak value of the inter-storey displacement over the duration of a seismic event. Therefore, we find the peak value using the following relation [32] between the mean of the peak response and the standard deviation of the process:

$$\mu_{Y_\tau} = p \cdot \sigma_Y, \tag{14}$$

where σ_Y is the standard deviation of the output response $y(t)$ (in the sense of ensemble average); Y_τ is the peak response of the absolute value of the output response $y(t)$ in a time period $(0,\tau]$; μ_{Y_τ} is the mean of Y_τ and p is the peak factor. The peak factor and the standard deviation are obtained by linear random vibration analysis of the structure with bi- or single-tuned mass damper subject to the Kanai–Tajimi PSD model.

The m-th order spectral moment of a stochastic process $y(t)$ is defined as

$$\lambda_m = \int_0^\omega \omega^m G_y(\omega)\,\mathrm{d}\omega, \tag{15}$$

where $G_y(\omega)$ is the one-sided PSD of the process $y(t)$. The variance of the output response is calculated by a frequency-domain random vibration analysis, i.e.,

$$\sigma_Y^2 = \lambda_0 = \int_0^\omega G_y(\omega)\,\mathrm{d}\omega = \int_0^\omega G_w(\omega)|H_{\mathrm{sys}}(\omega)|^2\,\mathrm{d}\omega, \tag{16}$$

where $G_w(\omega) = 2\Phi_{\ddot{x}_g\ddot{x}_g}(\omega)$ is the one-sided PSD of the stationary excitation, $H_{\mathrm{sys}}(\omega)$ is the frequency response function of the output response of interest. When we are interested in the inter-storey displacement of the structure, the corresponding output matrices $\mathbf{C_y}$ and $\mathbf{D_y}$ (see Eq. (5)) are

$$\mathbf{C_y} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix}, \quad \mathbf{D_y} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}. \tag{17}$$

Der Kiureghian [32] proposed a semi-empirical peak factor p that can take into account the effect of the bandwidth. For the two-sided thresholds, the peak factor is given as

$$p = \sqrt{2\ln(v_e\tau)} + \frac{0.5772}{\sqrt{2\ln(v_e\tau)}} \quad \text{for } v_e\tau \geqslant 2.1, \tag{18a}$$

$$p = 1.253 + 0.209 \times v_e\tau \quad \text{for } v_e\tau \leqslant 2.1, \tag{18b}$$

where τ is the duration of the stationary excitation, and v_e is given as

$$v_e = 2\delta v_y(0) \quad \text{for } 0<\delta\leqslant 0.1, \tag{19a}$$

$$v_e = (1.63\delta^{0.45} - 0.38) \times v_y(0) \quad \text{for } 0.1<\delta\leqslant 0.69, \tag{19b}$$

$$v_e = v_y(0) \quad \text{for } 0.69<\delta<1, \tag{19c}$$

where δ and $v_y(\zeta)$ represent the bandwidth parameter and the two-sided mean crossing rate of the process over the threshold ζ, respectively. These are given in terms of the spectral moments from Eq. (15) as

$$\text{Bandwidth of the process}: \ \delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0\lambda_2}}, \tag{20}$$

$$\text{Mean crossing rates}: \ v_y(\xi) = \frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}\exp\left(-\frac{\xi^2}{2\lambda_0}\right). \tag{21}$$

Thus, the mean peak value of the maximum inter-storey displacement of the 20-storey building structure with single- or bi-tuned mass dampers system can be estimated from Eqs. (14), (16) and (18). In order to

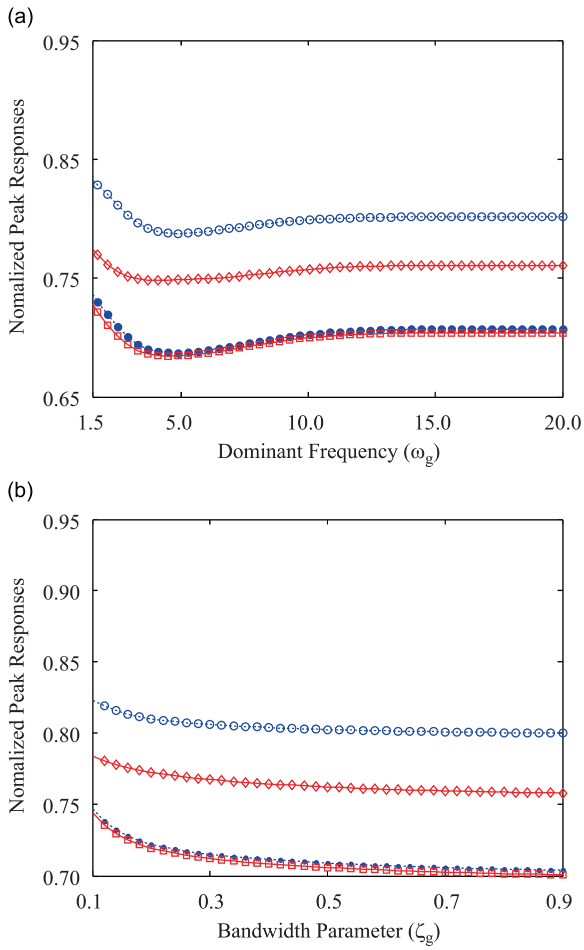

Fig. 8. Normalized peak responses of single-tuned mass damper and bi-tuned mass dampers systems for building A with the first modal frequency of 0.51 Hz with respect to variations in: (a) dominant frequency and (b) bandwidth parameter of the ground motions: ●: nominal performance of single-tuned mass damper by Den Hartog formula; □: nominal performance of bi-tuned mass dampers by proposed formula; ○: robust performance of single-tuned mass damper by Den Hartog formula; and ◇: robust performance of bi-tuned mass dampers by proposed formula.

evaluate the robust performances of the two damper systems, the inter-storey stiffness values of the building are varied from 85 to 115 percent of the nominal value. Fig. 8 presents the results for building A with the first modal frequency of 0.51 Hz. The performances of the optimal single- and bi-tuned mass dampers systems are compared for wide ranges of the Kanai–Tajimi parameter values in order to examine the performance of the optimal design for uncertain ground motion characteristics. Each of the peak responses of the two damper systems is normalized by the peak response of the original system with no damper in original condition (“nominal performance”) and in perturbed condition (“robust performance”), respectively. Fig. 8(a) shows the normalized responses of the damped building for a range of the dominant frequency of the ground motion, ω_g from 1.5 to 20 rad/s ($\zeta_g = 0.6$ and $\Phi_0 = 0.0871\,\mathrm{m^2/s^3}$). Since the vertical axis represents the ratio of the peak response of the damped system to that of the structure with no damper, the smaller value indicates more reduction in the peak response of the structure. Both systems show larger reduction ratio around the first modal frequency of the structural system, i.e., 3.20 rad/s. The improvement by the bi-tuned mass dampers is not significant in terms of nominal performance, whereas the bi-tuned mass dampers system shows a significant improvement in robust performance. Therefore, the bi-tuned mass dampers still remain effective in terms of nominal performance and further improve the robust performance. The effectiveness and robustness of the optimal bi-tuned mass dampers system hold the same for a range of bandwidth parameter ζ_g from 0.1 to

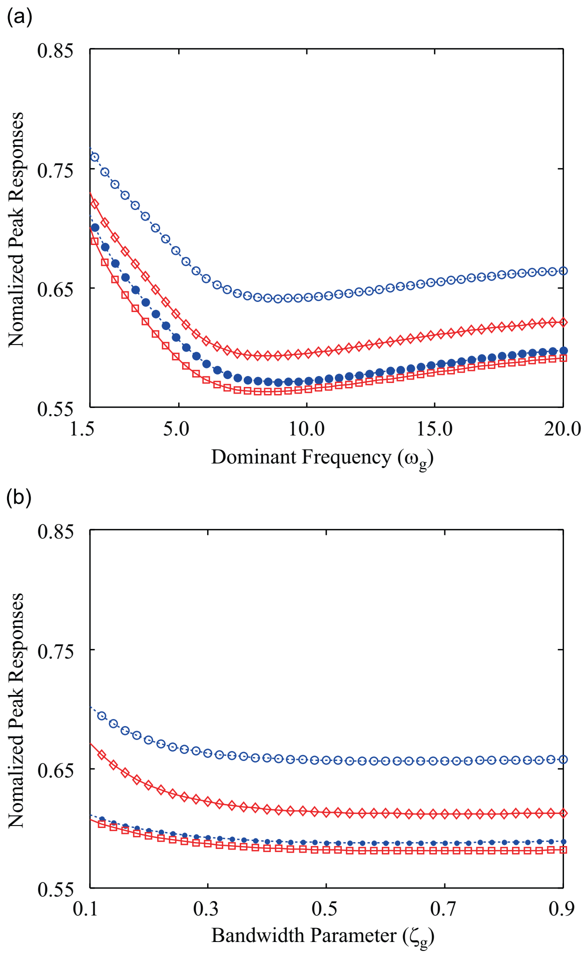

Fig. 9. Normalized peak responses of single-tuned mass damper and bi-tuned mass dampers systems for building B with the first modal frequency of 1.02 Hz with respect to variations in: (a) dominant frequency and (b) bandwidth parameter of the ground motions: ●: nominal performance of single-tuned mass damper by Den Hartog formula; □: nominal performance of bi-tuned mass dampers by proposed formula; ○: robust performance of single-tuned mass damper by Den Hartog formula; and ◇: robust performance of bi-tuned mass dampers by proposed formula.

0.9. ($\omega_g = 5\pi$ rad/s and $\Phi_0 = 0.0871\,\mathrm{m}^2/\mathrm{s}^3$). Fig. 8(b) plots the normalized responses versus the bandwidth parameter of the ground acceleration, ζ_g. The comparative results demonstrate that the bi-tuned mass dampers is superior in terms of the robust performances to the single-tuned mass damper system while keeping a similar level of nominal performance especially when the ground motion is relatively wide-band process. The effectiveness of the proposed design formula is confirmed for the example of building B for which the first modal frequency is 1.02 Hz (Fig. 9). In Fig. 9(a), the larger reduction appears again around the first model frequency of the building. The bi-tuned mass dampers system shows slightly improved nominal performance over single-tuned mass damper system while the robust performance of bi-tuned mass dampers still remains better than single-tuned mass damper. Similar results are also observed for the bandwidth of the ground motions as shown in Fig. 9(b).

6. Conclusion

In the study reported in this paper a closed-form, easy-to-use design formula for optimal performance of bi-tuned mass dampers system both in original and perturbed conditions was developed using non-dominated sorting genetic algorithm. The optimal parameters are identified using a multi-objective optimization and closed-formula is found by a nonlinear curve-fitting technique. In this study, the performance of the damped structural system is described by two measures, i.e., nominal and robust performance indices. The nominal performance index represents the effectiveness of the bi-tuned mass dampers when it is precisely tuned to the first modal frequency of the target structure, and the robust performance index stands for the robustness of the bi-tuned mass dampers against the frequency mis-tunings that arise from changes in dynamic properties of the target structure. In order to deal with the simultaneous optimization of the two performances, we adopt genetic algorithm-based multi-objective optimization method where the two competitive performance indices are used as a vector form of objective functions. This proposed approach systematically explores a set of Pareto-optimal solutions which are non-inferior or non-superior to each other in terms of the two objective functions. The Pareto-optimal surface in the objective function space shows the existence of a special bifurcation point which guarantees nominal performance with at least similar level to single-tuned mass damper and improves robust performance. The multi-objective optimization is repeated while varying the mass ratio of the bi-tuned mass dampers and an explicit design formula for the optimal bi-tuned mass dampers is derived using a nonlinear curve-fitting technique. The optimal tuning frequencies and damping ratios of the bi-tuned mass dampers are then expressed in terms of total mass ratio of the bi-tuned mass dampers. The accuracy of the proposed simple formula is verified through comparison between the performance indices by the formula and those obtained originally by multi-objective optimization. In order to demonstrate the applicability of the proposed design formula of bi-tuned mass dampers and evaluate their performances, illustrative examples are presented for optimal design problems of single- and bi-tuned mass dampers using full-order models of building structures. For the purpose of comparison, single-tuned mass damper systems designed by Den Hartog design formula are considered as well. Then, the nominal and robust performances of the optimal designs are examined by stochastic dynamic analysis for a wide range of uncertain characteristics of ground motions. The comparative results demonstrate that the proposed design formula can provide a bi-tuned mass dampers system that is both effective and robust in reducing the response of building structures under seismic excitations despite the uncertain characteristics of the stochastic ground motions and potential mis-tuning.

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